Vivekananda College of Engineering & Technology
[A Unit of Vivekananda Vidyavardhaka Sangha, Puttur ®]
Affiliated to Visvesvaraya Technological University
Approved by AICTE New Delhi & Govt of Karnataka

USN | | | | | | | | | | |

18MAT31

Third semester B.E Degree Preparatory examination February-2021

Transform Calculus, Fourier Series and Numerical Techniques

(Common to all Programmes)

Time: 3 hours Max. Marks: 100

Note: Answer FIVE full questions, choosing at least ONE question from each module

Module-1

1. (a) Find the Laplace transform of

(i) $\frac{e^{-at}-e^{-bt}}{t}$ (ii) $\left(\frac{4t+5}{e^{2t}}\right)^2$ (iii) $te^{-4t}\sin 3t$ 10 marks

(b) The square wave function f(t) with period 2a defined by $f(t)=\begin{cases}1 & 0\le t<a\\ -1 & a\le t<2a\end{cases}$

show that $L\{f(t)\}=\frac{1}{s}\tanh\left(\frac{as}{2}\right)$ 05 marks

(c) Solve by using Laplace transform $y''+4y'+4y=e^{-t}$ given that $y(0)=y'(0)=0$ 05 marks

OR

2. (a) Find (i) $L^{-1}\left\{\frac{s^2-3s+4}{s^3}\right\}$ (ii) $L^{-1}\left\{\frac{3s+2}{s^2-s-2}\right\}$ (iii) $L^{-1}\left\{\log\left(\frac{s+a}{s+b}\right)\right\}$ 10 marks

(b) Find $L^{-1}\left\{\frac{s}{(s-1)(s^2+4)}\right\}$ using convolution theorem 05 marks

(c) Express $f(t)=\begin{cases}2 & \text{if } 0<t<1\\ \frac{t^2}{2} & \text{if } 1<t<\frac{\pi}{2}\\ \cos t & \text{if } t>\frac{\pi}{2}\end{cases}$ in terms of unit step function and hence find its Laplace transforms 05 marks

Module-2

3. (a) Obtain the Fourier series of the function $f(x)=\begin{cases}2 & \text{if } -2<x<0\\ x & \text{if } 0<x<2\end{cases}$ 07 marks

(b) Find the half range sine series of $f(x)=e^x$ in (0, 1) 06 marks

(c) Obtain the Fourier series expansion of the function f(x) = |x| in $(-\pi, \pi)$ and hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\text{----------}=\frac{\pi^2}{8}$ 07 marks

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OR

4. (a) Obtain the Fourier series of the function f(x) = x for $0 \leq x \leq \pi$ and f(x) = 2π- x for $\pi \leq x \leq 2\pi$ and hence deduce the sum of the series $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} +$ ---------- = $\frac{\pi^2}{8}$ 07 marks

(b) Find the half range cosine series for the function f(x)= $(x-1)^2$ in $0 \leq x \leq 1$ 06 marks

(c) The following value of function y gives the displacement in inches of a certain machine part for rotations x of a flywheel. Expand y in terms of Fourier series up to second harmonics 07 marks

x	0	$\frac{\pi}{6}$	$\frac{2\pi}{6}$	$\frac{3\pi}{6}$	$\frac{4\pi}{6}$	$\frac{5\pi}{6}$	π
y	0	9.2	14.4	17.8	17.3	11.7	0

Module-3

5. (a) If f(x) = $\begin{cases} 1-x^2 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ Find the infinite Fourier transform of f(x) and hence Evaluate $\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right) \cos\left(\frac{x}{2}\right) dx$ 07 marks

(b) Find the Fourier cosine transform of f(x) = $\begin{cases} 4x & \text{if } 0 < x < 1 \\ 4-x & \text{if } 1 < x < 4 \\ 0 & \text{if } x > 4 \end{cases}$ 06 marks

(c) Solve $u_{n+2} - 3u_{n+1} + 2u_n = 3^n$ given $u_0 = u_1 = 0$ 07 marks

OR

6. (a) Find the Fourier Sine transform of $e^{-|x|}$ and hence show that $\int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m}$, m> 0 07 marks

(b) Find the z-transform of $u_n = \cosh\left(\frac{n\pi}{2} + \theta\right)$ 06 marks

(c) Find the inverse z-transform of $\frac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$ 07 marks

Module-4

7. (a) Find by Taylor's series method the value of y at x =0.1 and x=0.2 for the IVP $\frac{dy}{dx} = x^2y - 1; y(0) = 1$ 07 marks

(b) Use Runge Kutta method of fourth order to solve $\frac{dy}{dx} + y = 2x$ at x=1.1 given that y (1) =3. Take h =0.1 06 marks

(c) Given $\frac{dy}{dx} = xy + y^2$ with $y(0)=1$, $y(0.1)=1.1169$, $y(0.2)=1.2773$ $y(0.3)=1.5049$. Find $y(0.4)$ using Milne's Predictor Corrector method 07 marks

OR

8. (a) Given $\frac{dy}{dx} = x + \text{siny}$, $y(0)=1$ Compute $y(0.4)$ with h=0.2 using modified Euler's Method 07 marks

(b) Apply Runge Kutta method of fourth order to find an approximate value of y when x=0.5 given that $(x+y)\frac{dy}{dx} = 1$, y (0.4) =1. Take h=0.1 06 marks

(c) Using Adam Bashforth predictor corrector method find y (1.4) given that $\frac{dy}{dx} = x^2(1+y)$ with 07 marks

x	1	1.1	1.2	1.3
y	1	1.233	1.548	1.979

Module-5

9. (a) Given $y^{II} - y - xy^{I}=0$ with the initial conditions $y(0)= 1$, $y^{I}(0)=0$ compute $y(0.2)$, $y^{I}(0.2)$ using Runge Kutta method of Fourth order 07 marks

(b) Derive Euler's equation in the standard form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y^{I}}\right) = 0$ 06 marks

(c) Find the curves on which the functional $\int_0^1 \left(y^{I^2} + 12xy\right) dx$ with $y(0)=0$, $y(1)=1$ can be extremised 07 marks

OR

10. (a) Apply Milne's Predictor corrector method to compute $\frac{d^2y}{dx^2} = 1 + \frac{dy}{dx}$ and the following table of initial values 07 marks

x	0	0.1	0.2	0.3
y	1	1.1103	1.2427	1.3990
y^{I}	1	1.2103	1.4427	1.6990

(b) Find the extremal of the functional $\int_{x_1}^{x_2} \left(y^2 + y^{I\,2} + 2ye^x\right) dx$ 06 marks

(c) Prove that geodesics of a plane surface are straight lines 07 marks

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